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Light-like motions of particles in gravitational fields

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Abstract. Jaffe and Shapiro have recently shown that material particles (tardyons) in a spherically symmetric Schwarzschild field move in a light-like manner when their velocities at infinity are beyond a certain limit. We present here a more general discussion of the problem for different gravitational fields and extend our study to faster-than-light particles (tachyons).

1. Introduction

There are three kinds of particles—tardyons (material particles moving slower than light), luxons (massless particles like photons, neutrinos and gravitons travelling with the velocity of light) and tachyons (particles with imaginary mass moving faster than light). Recently Jaffe and Shapiro (1972) have shown that tardyons in a spherically symmetric Schwarzschild field move in a light-like manner§ when their velocities are beyond a certain limit. Here we have shown that tardyons exhibit light-like motions in different gravitational fields in regions depending upon their total energy. But it is found that tachyons have light-like motions throughout all static gravitational fields.

We proceed with our calculations in the following straightforward manner.

2. Tardyons

We know that the total energy of a material particle in a static field is given by (Landau and Lifshitz 1962)

$$E=\frac{m_0c^2\surd g_{44}}{\surd[1-(v^2/c^2)]}, \tag{1}$$

where

$$v^2=\left(\frac{c\,\mathrm{d}l}{\surd(g_{44})\,\mathrm{d}t}\right)^2, \tag{2}$$

$\mathrm{d}l$ being an invariant infinitesimal element of spatial displacement. We now define a velocity of the particle in terms of $\mathrm{d}l$ and $\surd(g_{44}(0))\,\mathrm{d}t$, the proper time of an observer

§ When particles move in a manner similar to photons in a field, they are said to behave in a light-like fashion.

O located at $x^\sigma(0)$ ($\sigma = 1, 2, 3$). Then, from (1) and (2),

$$\left(\frac{\mathrm{d}l}{\sqrt{(g_{44}(0))}\,\mathrm{d}t}\right)^2 = \frac{g_{44}}{g_{44}(0)}\left(1 - \frac{g_{44}}{K^2}\right), \tag{3}$$

where $K^2 = E/m_0c^2$. For a light ray this reduces to

$$\left(\frac{\mathrm{d}l}{\sqrt{(g_{44}(0))}\,\mathrm{d}t}\right)^2 = \frac{g_{44}}{g_{44}(0)}. \tag{4}$$

From (3) and (4) the condition for light-like motion of a tardyon is

$$\frac{\delta\{g_{44}[1-(g_{44}/K^2)]\}}{\delta g_{44}} > 0.$$

On simplification this reduces to:

$$K^2 > 2g_{44}. \tag{5}$$

A tardyon must satisfy this condition to have light-like motion in any static gravitational field.

2.1. *Schwarzschild field*

For a spherically symmetric gravitating body of mass m, the Schwarzschild metric is given as (using $c = G = 1$ henceforth)

$$\mathrm{d}s^2 = -\frac{\mathrm{d}r^2}{1-(2m/r)} - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) + \left(1 - \frac{2m}{r}\right)\mathrm{d}t^2. \tag{6}$$

In this field, the condition (5) reduces to

$$K^2 > 2\left(1 - \frac{2m}{r}\right). \tag{7}$$

Since the largest value of the right-hand side is 2 as $r \to \infty$ the condition for a particle to have light-like motion throughout the field is

$$K^2 > 2. \tag{8}$$

If v_0 is the velocity of the particle at infinity, then according to special relativity $K^2 = (1 - v_0^2)^{-1/2}$ so that (8) takes the form

$$v_0^2 > \tfrac{1}{2}. \tag{9}$$

This is the result obtained by Jaffe and Shapiro.

2.2. *Marder field*

The Marder metric (Marder 1958) for a cylindrically symmetric body is

$$\mathrm{d}s^2 = -A^2 r^{-2c^2-2c}(\mathrm{d}r^2 + \mathrm{d}z^2) - r^{2-2c}\,\mathrm{d}\phi^2 + r^{2c}\,\mathrm{d}t^2, \tag{10}$$

where $\frac{1}{2}c$ is the mass per unit length of the cylinder and A is a constant related to c and, for a cylinder of finite cross section, to the distribution of matter in the cylinder.

In this field, condition (5) reduces to

$$K^2 > 2r^{2c}. \tag{11}$$

This shows that particles move in a light-like manner in the Marder field in regions depending upon their total energy.

3. Tachyons

For a tachyon in a static field the expression for total energy takes the following form:

$$E = \frac{\mu c^2 \sqrt{g_{44}}}{\sqrt{[(v^2/c^2) - 1]}}, \tag{12}$$

where μ is related to the imaginary mass m_0 by

$$m_0 = \mathrm{i}\mu, \tag{13}$$

and v is given by (2). According to the definition we have adopted, the velocity of a tachyon is

$$\left(\frac{\mathrm{d}l}{\sqrt{(g_{44}(0))}\,\mathrm{d}t}\right)^2 = \frac{g_{44}}{g_{44}(0)}\left(1 + \frac{g_{44}}{K_1^2}\right), \tag{14}$$

where $K_1^2 = E/\mu c^2$. In this case, the condition corresponding to (5) is

$$K_1^2 > -2g_{44}. \tag{15}$$

Obviously this is always satisfied so that tachyons have light-like motions throughout all static gravitational fields.

One may easily extend these discussions to stationary fields and find that tardyons and tachyons exhibit light-like motions in these fields also.

References

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